Exam #3 - Practice

Instructions:

1. **Read** every problem CAREFULLY!
2. Make sure your answer addresses **each part** of the problem.
3. Make sure your answer is clearly marked by a circle or **box around the answer**.
4. Be sure to include the **proper units** and *3 significant figures* with your answer.
5. You may use a calculator for basic calculations.

Potentially Useful Information (continues on next page)

Constants

$k_B = 8.617 \times 10^{-5}$ eV/K

$= 1.381 \times 10^{-23}$ J/K

$h = 6.626 \times 10^{-34}$ J·s

$m_e = 9.11 \times 10^{-31}$ kg

$N_A = 6.022 \times 10^{23}$ atoms/mole

$R = 8.3145$ J/(K·mol)

$c = 3.00 \times 10^8$ m/s

$e = 1.60 \times 10^{-19}$ C

$(4\pi\varepsilon_0)^{-1} = 8.987 \times 10^9$ N·m^2·C^{-2}

1.00 eV $= 1.60 \times 10^{-19}$ J

Basics

atomic concentration = (mass density)×(Avagadro's number)/(atomic weight)

$$\rho = \frac{\sum(\text{Number of atoms of given type in unit cell} \times \text{Atomic mass of this type})}{\text{Volume of unit cell}}$$

$$PV = \left(\frac{N}{N_A}\right)RT = Nk_BT$$

$$L = L_0[1 + \lambda(T - T_0)]$$

$$n_v = N\exp\left(-\frac{E_v}{kT}\right)$$

$$D = D_o\exp\left(-\frac{E_A}{kT}\right)$$

$$L_{rms} = \sqrt{2Dt}$$

$$\rho = \rho_0[1 + \alpha_0\Delta T]$$

$$\rho_{alloy}\alpha_{alloy} = \rho_{pure}\alpha_{pure}$$

$$\rho_{alloy} = \rho_{pure} + CX(1-X)$$

$$P = IV = I^2R = V^2/R$$

$$V(r) = \frac{-1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

Quantum Mechanics

$KE_{max} = hf - \varphi$ $E_{ph} = hf = hc/\lambda$ $p = h/\lambda$ $\Delta x \Delta p \geq h/4\pi$

$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$ $E_n = \frac{h^2 n^2}{8 m_e a^2}$ $E_{n_x,n_y,n_z} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_x^2}{l_x^2} + \frac{n_y^2}{l_y^2} + \frac{n_z^2}{l_z^2}\right)$

Semiconductors

$f(E) = \frac{1}{1 + e^{\frac{E-E_F}{k_B T}}}$ Flux (Γ_{ph}) = #/(area×time) = Intensity/(hf) = Iλ/(hc)

$\sigma = en\mu_e + ep\mu_h$ $\lambda = v\tau$ $\tau_{e,h} = \frac{\mu_{e,h} m_{e,h}^*}{e}$ $m_e^* = \hbar^2\left(\frac{d^2E}{dk^2}\right)^{-1}$

$N_c = 2(\frac{2\pi m_e^* kT}{h^2})^{3/2}$ $N_v = 2(\frac{2\pi m_h^* kT}{h^2})^{3/2}$ $n_i = (N_c N_v)^{1/2} e^{-\frac{E_g}{2kT}}$

$np = n_i^2$

Key: element name / atomic number / **symbol** / atomic weight (mean relative mass)

1	2		3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
hydrogen 1 **H** 1.0079																		helium 2 **He** 4.0026
lithium 3 **Li** 6.941	beryllium 4 **Be** 9.0122												boron 5 **B** 10.811	carbon 6 **C** 12.011	nitrogen 7 **N** 14.007	oxygen 8 **O** 15.999	fluorine 9 **F** 18.998	neon 10 **Ne** 20.180
sodium 11 **Na** 22.990	magnesium 12 **Mg** 24.305												aluminium 13 **Al** 26.982	silicon 14 **Si** 28.086	phosphorus 15 **P** 30.974	sulfur 16 **S** 32.065	chlorine 17 **Cl** 35.453	argon 18 **Ar** 39.948
potassium 19 **K** 39.098	calcium 20 **Ca** 40.078		scandium 21 **Sc** 44.956	titanium 22 **Ti** 47.867	vanadium 23 **V** 50.942	chromium 24 **Cr** 51.996	manganese 25 **Mn** 54.938	iron 26 **Fe** 55.845	cobalt 27 **Co** 58.933	nickel 28 **Ni** 58.693	copper 29 **Cu** 63.546	zinc 30 **Zn** 65.38	gallium 31 **Ga** 69.723	germanium 32 **Ge** 72.61	arsenic 33 **As** 74.922	selenium 34 **Se** 78.96	bromine 35 **Br** 79.904	krypton 36 **Kr** 83.80
rubidium 37 **Rb** 85.468	strontium 38 **Sr** 87.62		yttrium 39 **Y** 88.906	zirconium 40 **Zr** 91.224	niobium 41 **Nb** 92.906	molybdenum 42 **Mo** 95.96	technetium 43 **Tc** [98]	ruthenium 44 **Ru** 101.07	rhodium 45 **Rh** 102.91	palladium 46 **Pd** 106.42	silver 47 **Ag** 107.87	cadmium 48 **Cd** 112.41	indium 49 **In** 114.82	tin 50 **Sn** 118.71	antimony 51 **Sb** 121.76	tellurium 52 **Te** 127.60	iodine 53 **I** 126.90	xenon 54 **Xe** 131.29
caesium 55 **Cs** 132.91	barium 56 **Ba** 137.33	57-70 *	lutetium 71 **Lu** 174.97	hafnium 72 **Hf** 178.49	tantalum 73 **Ta** 180.95	tungsten 74 **W** 183.84	rhenium 75 **Re** 186.21	osmium 76 **Os** 190.23	iridium 77 **Ir** 192.22	platinum 78 **Pt** 195.08	gold 79 **Au** 196.97	mercury 80 **Hg** 200.59	thallium 81 **Tl** 204.38	lead 82 **Pb** 207.2	bismuth 83 **Bi** 208.98	polonium 84 **Po** [209]	astatine 85 **At** [210]	radon 86 **Rn** [222]
francium 87 **Fr** [223]	radium 88 **Ra** [226]	89-102 **	lawrencium 103 **Lr** [262]	rutherfordium 104 **Rf** [267]	dubnium 105 **Db** [268]	seaborgium 106 **Sg** [271]	bohrium 107 **Bh** [272]	hassium 108 **Hs** [270]	meitnerium 109 **Mt** [276]	darmstadtium 110 **Ds** [281]	roentgenium 111 **Rg** [280]	ununbium 112 **Uub** [285]	ununtrium 113 **Uut** [284]	ununquadium 114 **Uuq** [289]	ununpentium 115 **Uup** [288]	ununhexium 116 **Uuh** [293]	ununseptium 117 **Uus** —	ununoctium 118 **Uuo** [294]

*lanthanoids	lanthanum 57 **La** 138.91	cerium 58 **Ce** 140.12	praseodymium 59 **Pr** 140.91	neodymium 60 **Nd** 144.24	promethium 61 **Pm** [145]	samarium 62 **Sm** 150.36	europium 63 **Eu** 151.96	gadolinium 64 **Gd** 157.25	terbium 65 **Tb** 158.93	dysprosium 66 **Dy** 162.50	holmium 67 **Ho** 164.93	erbium 68 **Er** 167.26	thulium 69 **Tm** 168.93	ytterbium 70 **Yb** 173.06
**actinoids	actinium 89 **Ac** [227]	thorium 90 **Th** 232.04	protactinium 91 **Pa** 231.04	uranium 92 **U** 238.03	neptunium 93 **Np** [237]	plutonium 94 **Pu** [244]	americium 95 **Am** [243]	curium 96 **Cm** [247]	berkelium 97 **Bk** [247]	californium 98 **Cf** [251]	einsteinium 99 **Es** [252]	fermium 100 **Fm** [257]	mendelevium 101 **Md** [258]	nobelium 102 **No** [259]

Problem 1: Carrier Statistics Revisited

Consider the valence electrons in the 3s-band of Mg at 300 K. The density of states in metals is given as

$$g(E) = (8\pi\sqrt{2})(\frac{m_e^*}{h^2})^{3/2}E^{1/2}$$

where the effective mass of electrons in the 3s-band of Mg is $1.30m_e$.

a. Since the band is roughly filled, integrate g(E) from the bottom of the band (E=0) to the top of the band (E_{3s}) to determine an expression for the total number of states within the band?

b. Given a value of E_{3s} of 11.5 eV (hint: use SI units for the calculation: $E_{3s} = 1.84 \times 10^{-18}$ J, then convert to cm^{-3}), what is the total number of states within the band (in cm^{-3})?

c. Given that Mg has a valence of 2 and a density of 1.738 g cm^{-3}, what is the concentration of valence electrons in Mg? (hint: the atomic weight for Mg is on the Periodic Table).

Now consider the electrons in the conduction band of intrinsic Si at 300 K. The conduction band electron density of states, N_c, and concentration, n, are given, respectively by

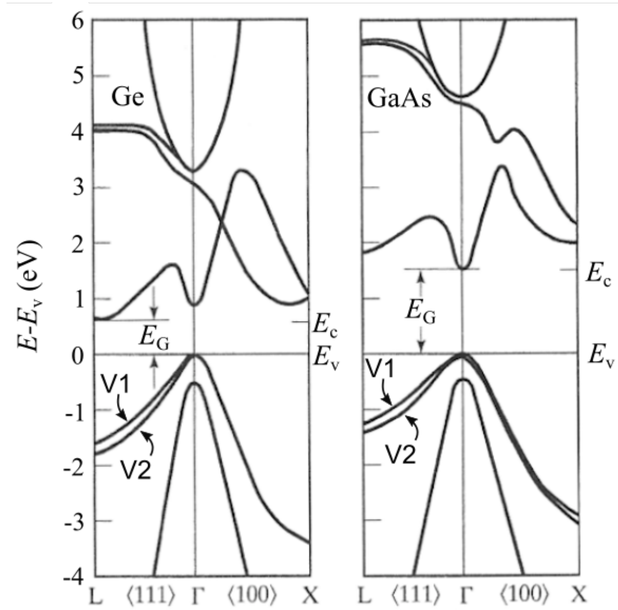
$$N_c = 2(\frac{2\pi m_e^* kT}{h^2})^{3/2} \quad \text{and} \quad n = N_c e^{-\frac{E_C - E_F}{kT}}.$$

d. Given the density of states effective mass of conduction band electrons in Si of $1.08m_e$, what is the value of the density of states, N_c, in the conduction band of Si at 300 K (in cm^{-3})?

e. The band gap of Si is 1.10 eV. What is electron concentration (in cm^{-3}) in the conduction band of Si at 300 K (hint: assume the Fermi energy is mid gap)?

f. Based on your results from above, which statistical description (Maxwell-Boltzmann statistics or Fermi-Dirac statistics) should we use for electrons in Mg and for Si? Explain your answer by considering the probability of electrons trying to occupy the same states and the Pauli Exclusion principle.

Problem 2: *E-k* Plots



Consider the room temperature (300 K) energy versus wave vector (E-k) plots shown for Ge and GaAs.

a. In precise, specific language, how does one deduce the value of the band gap, E_G, from the diagrams?

b. What is the numerical value of E_G for each material?

c. In precise, specific language, explain how the E-k plots can be used to determine whether a semiconductor is a direct or indirect band gap semiconductor.

d. Are Ge and GaAs direct or indirect band gap semiconductors?

e. Precisely how does one use an E-k relationship to determine the effective masses for carriers in a semiconductor? (hint: there is definitely one key word your answer should include and it's not "slope")

f. Are effective masses the same for electrons and holes?

g. Relative to the mass of the free electron m_e, the effective masses for GaAs are given as

$m_e^* = 0.067m_e$ for the electrons in the conduction band

$m_{hh}^* = 0.51m_e$ for the heavy holes in the valence band

$m_{lh}^* = 0.082m_e$ for the light holes in the valence band

As best as you can estimate, are these values consistent with the E-k plot?

h. Which bands are the "heavy holes" and which are the "light holes"?

i. Since E is proportional to k^2, one might expect the E-k relationships to be symmetric about the zone center (Γ point, $k = 0$). Why are these E-k plots not symmetric about Γ?

Problem 3: Intrinsic and Extrinsic Semiconductors

For the III-V semiconductor GaAs, $E_g = 1.42$ eV, $N_c = 4.40\times10^{17}$ cm^{-3}, $N_v = 7.70\times10^{18}$ cm^{-3}, $\mu_e = 8800$ cm^2 V^{-1} s^{-1}, $\mu_h = 400$ cm^2 V^{-1} s^{-1}, $m_e^* = 0.067m_e$, and $m_h^* = 0.500m_e$.

a. Calculate the intrinsic carrier concentration (in cm^{-3}) at 300 K.

b. Calculate the intrinsic resistivity (in Ω cm) at 300 K.

c. Now consider doping GaAs with Si. If we want an p-type GaAs, which element must Si replace in GaAs?

d. If we dope GaAs with Si to a p-type dopant concentration of $N_a = 4.41\times10^{12}$ cm^{-3}, what is the resulting electron concentration (in cm^{-3}) and resistivity (in Ω cm) at 300 K?

e. If instead we doped GaAs with Si to an n-type dopant concentration of $N_d = 4.41\times10^{12}$ cm^{-3}, will its resistivity be higher, lower, or the same as you found in part (d)? Can you answer this without calculating the resistivity explicitly? Explain.

f. (Extra credit) Now consider an intrinsic (undoped) GaAs crystal at 300K that is being illuminated with 1.00 second long pulses from a Ti:sapphire laser (λ = 800 nm). The crystal has dimensions of 1 cm × 1 cm × 1 cm, and the incident light is uniformly and completely absorbed over the volume of the crystal. Assuming that each absorbed photon creates an electron-hole pair, what laser intensity (in μW/cm^2) is required such that the resistivity of the illuminated intrinsic GaAs will equal the resistivity of the doped GaAs from part (d)?