Homework #9 - Solutions

Problems

1. **Temperature Dependence of Conductivity (Kasap 5.12)**

   An n-type Si sample has been doped with 10^{15} phosphorus atoms cm^{-3}. The donor energy level for P in Si is 0.045 eV below the conduction band edge energy.

   a. Calculate the room temperature conductivity of the sample.
   b. Estimate the temperature above which the sample behaves as if intrinsic.
   c. Estimate to within 20 percent the lowest temperature above which all the donors are ionized.
   d. Sketch schematically the dependence of the electron concentration in the conduction band on the temperature as $\log(n)$ versus $1/T$, and mark the various important regions and critical temperatures. For each region draw an energy band diagram that clearly shows from where the electrons are excited into the conduction band.
   e. Sketch schematically the dependence of the conductivity on the temperature as $\log(\sigma)$ versus $1/T$ and mark the various critical temperatures and other relevant information..

Solution

a. At room temperature (T = 300 K), $\mu_e = 1400 \times 10^{-4}$ m^2 V^{-1} s^{-1} from Table 5.1. The conductivity temperature T = 300 K is

$$\sigma = eN_d\mu_e$$

$\therefore$ $$\boldsymbol{\sigma} = (1.602 \times 10^{-19}\text{ C})(1 \times 10^{21}\text{ m}^{-3})(1400 \times 10^{-4}\text{ m}^2\text{ V}^{-1}\text{ s}^{-1}) = \mathbf{22.4\ \Omega^{-1}\ m^{-1}}$$

b. At $T = T_i$, the intrinsic concentration $n_i = N_d = 1 \times 10^{15}$ cm^{-3}. From Figure 5.16, as shown in Figure 5Q12-1, the graph of $n_i(T)$ versus $1/T$,

$$1000 / T_i = 1.85\text{ K}^{-1}$$

$\therefore$ $$\boldsymbol{T_i} = 1000 / (1.85\text{ K}^{-1}) = \mathbf{540\ K\ or\ 267\ °C}$$

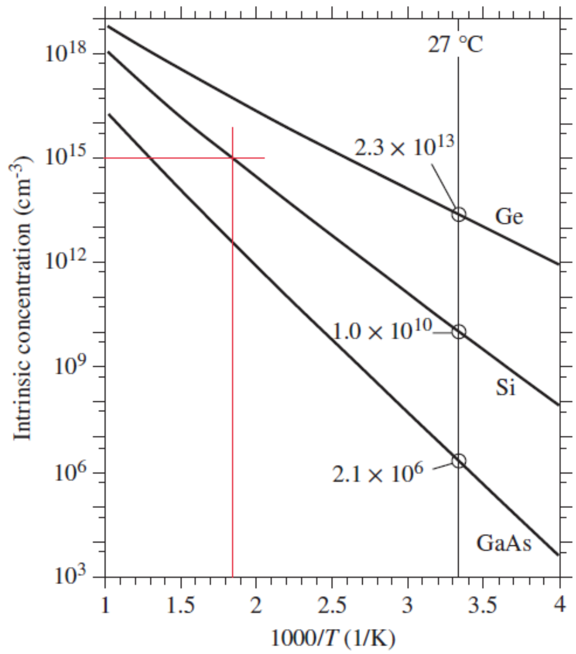

Figure 5Q12-1 Figure 5.16, the temperature dependence of the intrinsic concentration. At $n_i = 10^{15}$ cm^{-3}, $1000/T_i = 1.85$.

c. The ionization region ends at $T = T_s$ when all donors have been ionized, *i.e.* when $n = N_d$. From Example 5.8, at $T = T_s$:

$$n = N_d = \left(\frac{1}{2}N_cN_d\right)^{\frac{1}{2}}\exp\left(\frac{-\Delta E}{2kT_s}\right)$$

$$\therefore \qquad T_s = \frac{-\Delta E}{2k\ln\left(\frac{N_d}{\sqrt{\frac{1}{2}N_cN_d}}\right)} = \frac{-\Delta E}{2k\ln\left(\sqrt{\frac{2N_d}{N_c}}\right)}$$

$$\therefore \qquad T_s = \frac{\Delta E}{k\ln\left(\frac{N_c}{2N_d}\right)}$$

Take $N_c = 2.8 \times 10^{19}$ cm^{-3} at 300 K from Table 5.1, and the difference between the donor energy level and the conduction band energy is $\Delta E = 0.045$ eV. Therefore our first approximation to T_s is:

$$T_s = \frac{\Delta E}{k\ln\left(\frac{N_c}{2N_d}\right)} = \frac{(0.045\text{ eV})(1.602\times10^{-19}\text{ J/eV})}{(1.381\times10^{-23}\text{ J/K})\ln\left(\frac{(2.8\times10^{19}\text{ cm}^{-3})}{2(10^{15}\text{ cm}^{-3})}\right)} = 54.68\text{ K}$$

Find the new N_c at this temperature, N_c':

$$N_c' = N_c\left(\frac{T_s}{300}\right)^{\frac{3}{2}} = \left(2.8\times10^{19}\ \text{cm}^{-3}\right)\left(\frac{54.68\ \text{K}}{300\ \text{K}}\right)^{\frac{3}{2}} = 2.179\times10^{18}\ \text{cm}^{-3}$$

Find a better approximation for T_s by using this new N_c',

$$T_s' = \frac{\Delta E}{k\ln\left(\frac{N_c'}{2N_d}\right)} = \frac{\left(0.045\ \text{eV}\right)\left(1.602\times10^{-19}\ \text{J/eV}\right)}{\left(1.381\times10^{-23}\ \text{J/K}\right)\ln\left(\frac{\left(2.179\times10^{18}\ \text{cm}^{-3}\right)}{2\left(10^{15}\ \text{cm}^{-3}\right)}\right)} = 74.64\ \text{K}$$

$$\therefore \quad N_c'' = N_c\left(\frac{T_s'}{300}\right)^{\frac{3}{2}} = \left(2.8\times10^{19}\ \text{cm}^{-3}\right)\left(\frac{74.64\ \text{K}}{300\ \text{K}}\right)^{\frac{3}{2}} = 3.475\times10^{18}\ \text{cm}^{-3}$$

A better approximation to T_s is:

$$T_s'' = \frac{\Delta E}{k\ln\left(\frac{N_c''}{2N_d}\right)} = \frac{\left(0.045\ \text{eV}\right)\left(1.602\times10^{-19}\ \text{J/eV}\right)}{\left(1.381\times10^{-23}\ \text{J/K}\right)\ln\left(\frac{\left(3.475\times10^{18}\ \text{cm}^{-3}\right)}{2\left(10^{15}\ \text{cm}^{-3}\right)}\right)} = 69.97\ \text{K}$$

$$\therefore \quad N_c''' = N_c\left(\frac{T_s''}{300}\right)^{\frac{3}{2}} = \left(2.8\times10^{19}\ \text{cm}^{-3}\right)\left(\frac{69.97\ \text{K}}{300\ \text{K}}\right)^{\frac{3}{2}} = 3.154\times10^{18}\ \text{cm}^{-3}$$

$$\therefore \quad T_s''' = \frac{\Delta E}{k\ln\left(\frac{N_c'''}{2N_d}\right)} = \frac{\left(0.045\ \text{eV}\right)\left(1.602\times10^{-19}\ \text{J/eV}\right)}{\left(1.381\times10^{-23}\ \text{J/K}\right)\ln\left(\frac{\left(3.154\times10^{18}\ \text{cm}^{-3}\right)}{2\left(10^{15}\ \text{cm}^{-3}\right)}\right)} = 70.89\ \text{K}$$

We can see that the change in T_s is very small, and for all practical purposes we can consider the calculation as converged. Therefore $\boldsymbol{T_s = 70.9\ \text{K} = -202.1\ °\text{C}}$.

d. and ***e.***

See Figures 5.15 and 5.20

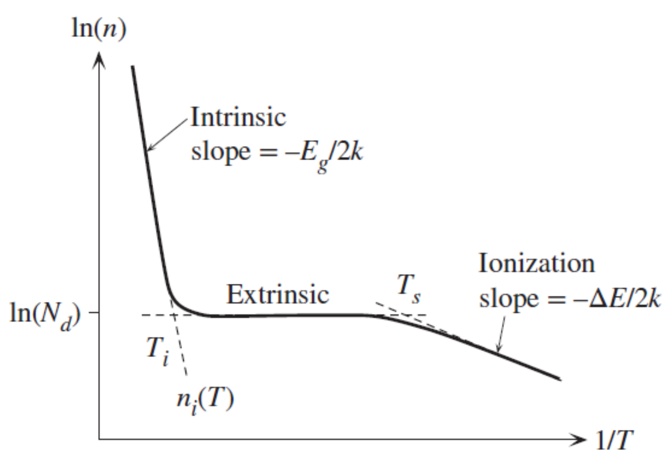

Figure 5.15: The temperature dependence of the electron concentration in an *n*-type semiconductor.

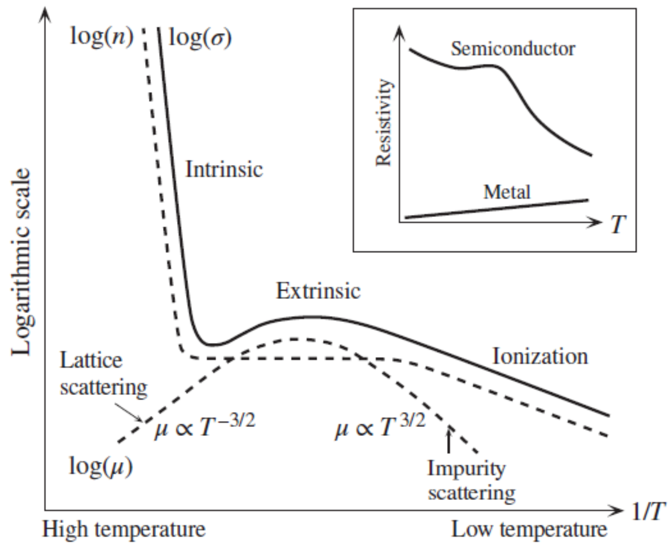

Figure 5.20: Schematic illustration of the temperature dependence of electrical conductivity for a doped (*n*-type) semiconductor.

2. **Degenerate Semiconductor (Kasap 5.21)**

Consider the general exponential expression for the concentration of electrons in the CB,

$$n = N_c \exp\left[-\frac{(E_c - E_F)}{kT}\right]$$

and the mass action law, $np = n_i^2$. What happens when the doping level is such that n approaches N_c and exceeds it? Can you still use the above expressions for n and p?

Consider an n-type Si that has been heavily doped and the electron concentration in the CB is 10^{20} cm^{-3}. Where is the Fermi level? Can you use $np = n_i^2$ to find the hole concentration? What is its resistivity? How does this compare with a typical metal? What use is such a semiconductor?

Solution

Consider $n = N_c\exp[-(E_c - E_F)/kT]$ (1)

and $np = n_i^2$ (2)

These expressions have been derived using the Boltzmann tail ($E > E_F$ + a few kT) to the Fermi – Dirac (FD) function $f(E)$ as in Section 5.1.4. Therefore the expressions are NOT valid when the Fermi level is within a few kT of E_c. In these cases, we need to consider the behavior of the FD function $f(E)$ rather than its tail and the expressions for n and p are complicated.

It is helpful to put the 10^{20} cm^{-3} doping level into perspective by considering the number of atoms per unit volume (atomic concentration, n_{at}) in the Si crystal. From Appendix C, the density is 2.33 g cm^{-3}, so that

$$n_{\text{at}} = \frac{(\text{Density})N_A}{M_{\text{at}}} = \frac{(2.33\times10^{3}\,\text{kg m}^{-3})(6.022\times10^{23}\,\text{mol}^{-1})}{(28.09\times10^{-3}\,\text{kg mol}^{-1})}$$

i.e. $n_{at} = 4.995 \times 10^{28}$ m^{-3} or 5.0×10^{22} cm^{-3}

Given that the electron concentration $n = 10^{20}$ cm^{-3}, we see that

$$n/n_{at} = (10^{20}\ \text{cm}^{-3}) / (5.0 \times 10^{22}\ \text{cm}^{-3}) = 0.00200$$

which means that if all donors could be ionized we would need 1 in 500 doping or 0.2% donor doping in the semiconductor. We cannot use Equation (1) to find the position of E_F. The Fermi level will be in the conduction band. The semiconductor is degenerate. The electrical properties are more like that of a metal than that of a semiconductor. The conductivity would be very high compared to a nondegenerate semiconductor. Highly doped (degenerate) semiconductor regions in devices can replace metals by providing sufficiently high conductivity, and without needing to deposit actual metals. Degenerate semiconductors are widely used in a variety of devices, as will be apparent in Chapter 6.

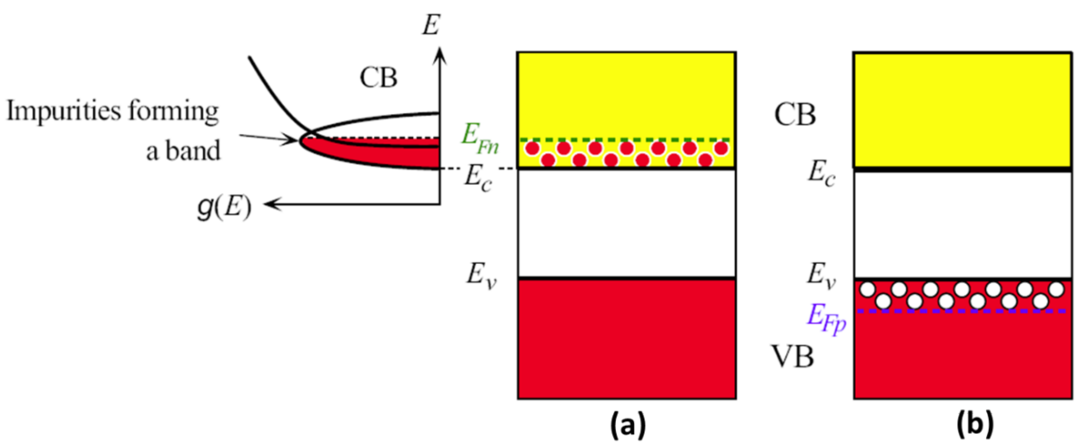

Figure 5.21 (a) Degenerate *n*-type semiconductor. Large number of donors form a band that overlaps the CB. (b) Degenerate *p*-type semiconductor.