Homework #8 - Solutions

Problem 1 - Bandgaps and Photodetection

a. Determine the maximum value of the energy gap that a semiconductor, used as a photoconductor, can have if it is to be sensitive to yellow light (600 nm).

We’re given the wavelength of incident photons. The energy of the photons must be equal to or greater than the bandgap. Thus, the maximum bandgap will equal the photon energy.

$$E_g = Eph = \frac{hc}{\lambda} = \frac{1241\ \text{eV nm}}{(600\ \text{nm})} = 2.07\ \text{eV}$$

$\boxed{\text{The maximum bandgap is 2.07 eV.}}$

b. A photodetector whose area is 5×10^{-2} cm^2 is irradiated with yellow light whose intensity is 2 mW cm^{-2}. Assuming that each photon generates one electron-hole pair, calculate the number of pairs generated per second.

If the intensity is $2\times10^{-3}\ W/\text{cm}^2$ and the area is 5×10^{-2} cm^2, then the total incident power is 10^{-4} W. The number of incident photons is equal to the incident power divided by the energy per photon (in Joules):

$$N_{ph} = \frac{P}{E_{ph}} = \frac{10^{-4}\ \text{J/s}}{3.31\times10^{-19}\ \text{J}} = 3.02\times10^{14}\ \text{photons/s}$$

Assuming that each photon generates an electron – hole pair,

then the number of e – h pairs is $\boxed{N_{ehp} = 3.02\times10^{14}\ \text{s}^{-1}}$

c. From the known energy gap of the semiconductor GaAs (E_g = 1.42 eV), calculate the primary wavelength of photons emitted from this crystal as a result of electron-hole recombination.

For GaAs, the primary wavelength of emitted photons is given by the bandgap energy.

$$\lambda_{GaAs} = \frac{hc}{E_{ph}} = \frac{hc}{E_g} = \frac{1241\ \text{eV nm}}{1.42\ \text{eV}} = \boxed{874\ \text{nm}}$$

d. What part of the electromagnetic spectrum is the wavelength from part (c)?

The wavelength from part (c) is in the near infrared portion of the spectrum.

e. Will a silicon photodetector be sensitive to the radiation from a GaAs laser? Why?

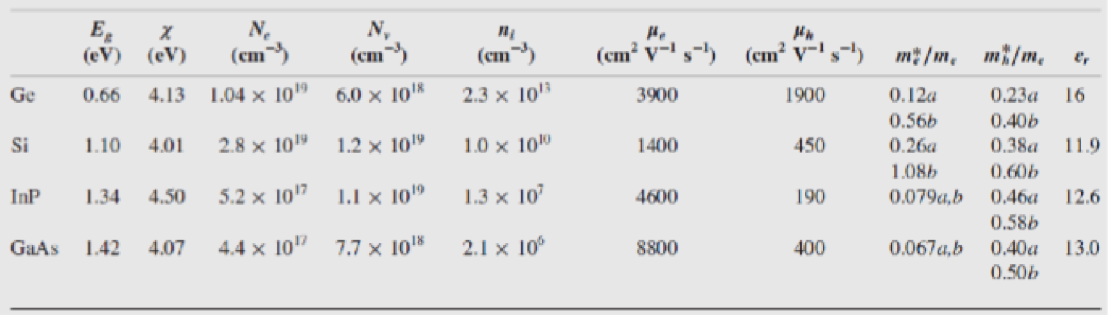
The bandgap of Si is 1.10 eV, so the maximum photon wavelegth that can be detected by a Si photodetector is roughly.

$$\lambda_{Si} = \frac{hc}{E_{ph}} = \frac{hc}{E_g} = \frac{1241\ \text{eV nm}}{1.10\ \text{eV}} = \boxed{1128\ \text{nm}}$$

Since the wavelength emitted by the GaAs laser is 874 nm, which is shorter than the maximum wavelength of 1128 nm for Si. $\boxed{\text{Yes, the Si photodetector can detect the emission of a GaAs laser.}}$

Problem 2 - Intrinsic Ge

Table 5.1 Selected typical properties of Ge, Si, InP, and GaAs at 300 K



	E_g (eV)	χ (eV)	N_c (cm^{-3})	N_v (cm^{-3})	n_i (cm^{-3})	μ_e ($cm^2 V^{-1} s^{-1}$)	μ_h ($cm^2 V^{-1} s^{-1}$)	m_e^*/m_e	m_h^*/m_e	ε_r
Ge	0.66	4.13	1.04×10^{19}	6.0×10^{18}	2.3×10^{13}	3900	1900	0.12*a* 0.56*b*	0.23*a* 0.40*b*	16
Si	1.10	4.01	2.8×10^{19}	1.2×10^{19}	1.0×10^{10}	1400	450	0.26*a* 1.08*b*	0.38*a* 0.60*b*	11.9
InP	1.34	4.50	5.2×10^{17}	1.1×10^{19}	1.3×10^{7}	4600	190	0.079*a,b*	0.46*a* 0.58*b*	12.6
GaAs	1.42	4.07	4.4×10^{17}	7.7×10^{18}	2.1×10^{6}	8800	400	0.067*a,b*	0.40*a* 0.50*b*	13.0

a. Using the values of the density of states effective masses m_e^* and m_h^* in Table 5.1, calculate the intrinsic concentration in Ge.

From Table 5.1, we get

```
In[∘]:= me = 9.11×10^-31 (* kg *);
        meffe = 0.56 me
        meffh = 0.4 me

Out[∘]=
        5.1016×10^-31

Out[∘]=
        3.644×10^-31
```

With these, we have

```
In[∘]:= kB = 1.38×10^-23 (* J/K *);
        Temp = 300 (* K *);
        h = 6.626×10^-34 (* J s *);
        Nc = 2 ((2 π meffe kB Temp)/h^2)^(3/2)
        Nv = 2 ((2 π meffh kB Temp)/h^2)^(3/2)

Out[∘]=
        1.05101×10^25

Out[∘]=
        6.34474×10^24
```

These densities of states are in Si units of m^{-3}. I'll convert to cm^{-3} in the next line by dividing by 10^6.

In[∘]:= $$\text{ni} = \sqrt{\text{Nc Nv}}\ \text{Exp}\left[\frac{-0.66}{2\times 8.62\times 10^{-5}\times 300}\right] 10^{-6}$$

Out[∘]= $$2.34406\times 10^{13}$$

The intrinsic carrier concentration in Ge at 300 *K* is $2.34*10^{13}$ cm^{-3}.

b. What is n_i if you use N_c and N_v from Table 5.1?

From Table 5.1, $N_c = 1.04\times10^{19}$ cm^{-3} and $N_v = 6.00\times10^{18}$ cm^{-3}. Using these in the equation for n_i, we have

In[∘]:= $$\text{ni2} = \sqrt{1.04\times 10^{19}\times 6\times 10^{18}}\ \text{Exp}\left[\frac{-0.66}{2\times 8.62\times 10^{-5}\times 300}\right]$$

Out[∘]= $$2.26752\times 10^{13}$$

Using the Table 5.1 densities of states values,
the intrinsic carrier concentration is nearly the same at 2.27×10^{13} cm^{-3}.

c. Calculate the intrinsic resistivity of Ge at 300 K .

Since n = p for intrinsic semiconductors, the intrinsic resistivity is $\rho = \sigma^{-1} = [en\mu_e + ep\mu_h]^{-1} = [en_i(\mu_e + \mu_h)]^{-1}$

In[∘]:= $$\text{rhoi} = \left(1.6\times 10^{-19}\ \text{ni}\ (3900+1900)\right)^{-1}$$

Out[∘]= $$45.9709$$

The intrinsic resistivity is 46.0 Ω cm. Your values could be slightly different if you used the intrinsic concentration from part b.

Problem 3 - Extrinsic Si

a. A Si crystal has been doped with P. The donor concentration is 10^{15} cm^{-3}. Find the conductivity, and resistivity of the crystal. Use Kasap Table 5.1 (above) for carrier mobility.

We are given a dopant concentration of 10^{15} cm^{-3}, which is five orders of magnitude greater than the intrinsic concentration. Since P is a group V element, it is an n-type dopant in Si and provides excess electrons. Thus, the conductivity and resistivity of the crystal are

$$\sigma = eN_d\,\mu_e \quad \text{and} \quad \rho = 1/\sigma$$

In[1]:= $$\text{sigma} = 1.6\times 10^{-19}\ (*\text{C}*)\ 10^{15}\ (*\text{cm}^{-3}*)\ 1350\ (*\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}*)$$

Out[1]= 0.216

In[2]:= rho = 1 / sigma

Out[2]= 4.62963

The units of sigma are Ω^{-1} cm^{-1} and the units of rho are Ω cm.

Thus, for the doped Si,
the extrinsic conductivity is 0.216 $\Omega^{-1}\ \text{cm}^{-1}$ and the extrinsic resistivity is 4.63 Ω cm.

You may be wondering about why we neglected the hole contribution to the conductivity. We can use the mass action law to determine the concentration of holes:

$$pn = n_i^2 \Rightarrow p = \frac{n_i^2}{n} = \frac{n_i^2}{N_d}$$

In[3]:= $\textbf{holes} = \frac{(10^{10})^2}{10^{15}}$

Out[3]= 100 000

Since the hole concentration $(10^5\ \text{cm}^{-3})$ is 10 orders of magnitude lower than the electron concentration, the hole contribution to the conductivity is negligible.

Homework #8 - Solutions

Problems 4-5

4. Extrinsic Si

a. Find the concentration of acceptors required for a *p*-type Si crystal to have a resistivity of 1 Ω cm. Use Kasap Table 5.1 (above) for carrier mobility.

b. Near the doping concentration found in part (a), the carrier mobility is closer 350 cm^2 V^{-1} s^{-1} (see Fig. 5.19 in Kasap). With this mobility, what is the necessary acceptor concentration?

Solution

The resistivity, $\rho = \dfrac{1}{eN_a\mu_h}$

$$\therefore \qquad N_a = \frac{1}{e\mu_h\rho} = \frac{1}{(1.6\times10^{-19}\,\text{C})(450\,\text{cm}^2\text{V}^{-1}\text{s}^{-1})(1\,\Omega\,\text{cm})} = \mathbf{1.38\times10^{16}\ cm^{-3}}$$

Above, it was assumed that the doping concentration does not affect the drifty mobility. At this concentration, from Figure 5.19 (Kasap 4th, pg. 443), the hole drift mobility is roughly 350 cm^2 V^{-1} s^{-1} rather than 450 cm^2 V^{-1} s^{-1}. Using 350 cm^2 V^{-1} s^{-1} we find,

$$N_a = \mathbf{1.79\times10^{16}\ cm^{-3}}$$

5. Minimum conductivity

a. Consider the conductivity of a semiconductor, $\sigma = en\mu_e + ep\mu_h$. Will doping always increase the conductivity?

b. Show that the minimum conductivity for Si is obtained when it is *p*-type doped such that the hole concentration is

$$p_m = n_i\sqrt{\frac{\mu_e}{\mu_h}}$$

and the corresponding minimum conductivity (maximum resistivity) is

$$\sigma_{\min} = 2en_i\sqrt{\mu_e\mu_h}$$

c. Calculate p_m and $\sigma_{\min}$ for Si and compare with intrinsic values.

Solution

***a.* Doping does *not* always increase the conductivity**. Suppose that we have an intrinsic sample with $n = p$ but the hole drift mobility is smaller. If we dope the material very slightly with *p*-type then $p > n$. However, this would decrease the conductivity because it would create more holes with lower mobility at the expense of electrons with higher mobility. Obviously with further doping p increases sufficiently to result in the conductivity increasing with the extent of doping.

b. To find the minimum conductivity, first consider the mass action law:

$$np = n_i^2$$

isolate n: $n = n_i^2/p$

Now substitute for n in the equation for conductivity:

$$\sigma = en\mu_e + ep\mu_h$$

$$\therefore \qquad \sigma = \frac{en_i^2\mu_e}{p} + \mu_h ep$$

To find the value of p that gives minimum conductivity (p_m), differentiate the above equation with respect to p and set it equal to zero:

$$\frac{d\sigma}{dp} = -\frac{en_i^2\mu_e}{p^2} + \mu_h e$$

$$\therefore \qquad -\frac{en_i^2\mu_e}{p_m^2} + \mu_h e = 0$$

Isolate p_m and simplify,

$$p_m = n_i\sqrt{\frac{\mu_e}{\mu_h}}$$

Substituting this expression back into the equation for conductivity will give the minimum conductivity:

$$\sigma_{\min} = \frac{en_i^2\mu_e}{p_m} + \mu_h ep_m = \frac{en_i^2\mu_e}{n_i\sqrt{\mu_e/\mu_h}} + \mu_h en_i\sqrt{\frac{\mu_e}{\mu_h}}$$

$$\therefore \qquad \sigma_{\min} = en_i\mu_e\sqrt{\frac{\mu_h}{\mu_e}} + en_i\sqrt{\mu_e\mu_h} = en_i\sqrt{\mu_e\mu_h} + en_i\sqrt{\mu_e\mu_h}$$

$$\therefore \qquad \sigma_{\min} = 2en_i\sqrt{\mu_e\mu_h}$$

c. From Table 5.1, for Si: $\mu_e = 1350$ cm^2 V^{-1} s^{-1}, $\mu_h = 450$ cm^2 V^{-1} s^{-1} and $n_i = 1.0 \times 10^{10}$ cm^{-3}. Substituting into the equations for p_m and $\sigma_{\min}$:

$$p_m = n_i\sqrt{\frac{\mu_e}{\mu_h}} = \left(1.0\times10^{10}\ \text{cm}^{-3}\right)\sqrt{\frac{1350\ \text{cm}^2\ \text{V}^{-1}\ \text{s}^{-1}}{450\ \text{cm}^2\ \text{V}^{-1}\ \text{s}^{-1}}} \ \mathbf{= 1.73 \times 10^{10}\ cm^{-3}}$$

$$\sigma_{\min} = 2en_i\sqrt{\mu_e\mu_h}$$

$$\therefore \qquad \sigma_{\min} = 2\left(1.602\times10^{-19}\ \text{C}\right)\left(1.0\times10^{10}\ \text{cm}^{-3}\right)\sqrt{\left(1350\ \text{cm}^2\ \text{V}^{-1}\ \text{s}^{-1}\right)\left(450\ \text{cm}^2\ \text{V}^{-1}\ \text{s}^{-1}\right)}$$

$\therefore \qquad \boldsymbol{\sigma_{\text{min}} = 2.5 \times 10^{-6}\ \Omega^{-1}\ \text{cm}^{-1}}$

The corresponding maximum resistivity is:

$$\rho_{\text{max}} = 1 / \sigma_{\text{min}} = \mathbf{4 \times 10^5\ \Omega\ cm}$$

The intrinsic value corresponding to p_m is simply n_i (= 1.0×10^{10} cm^{-3}). Comparing it to p_m:

$$\frac{p_m}{n_i} = \frac{1.73 \times 10^{10}\ \text{cm}^{-3}}{1.0 \times 10^{10}\ \text{cm}^{-3}} = 1.73$$

The intrinsic conductivity is:

$$\sigma_{\text{int}} = en_i(\mu_e + \mu_h)$$

$\therefore \qquad \sigma_{\text{int}} = (1.602 \times 10^{-19}\ \text{C})(1.0 \times 10^{10}\ \text{cm}^{-3})(1350\ \text{cm}^2\ \text{V}^{-1}\ \text{s}^{-1} + 450\ \text{cm}^2\ \text{V}^{-1}\ \text{s}^{-1})$

$\therefore \qquad \sigma_{\text{int}} = 2.88 \times 10^{-6}\ \Omega^{-1}\ \text{cm}^{-1}$

Comparing this value to the minimum conductivity:

$$\frac{\sigma_{\text{min}}}{\sigma_{\text{int}}} = \frac{2.5 \times 10^{-6}\ \Omega^{-1}\ \text{cm}^{-1}}{2.88 \times 10^{-6}\ \Omega^{-1}\ \text{cm}^{-1}} = 0.87$$

Sufficient *p*-type doping that increases the hole concentration by 73% *decreases* the conductivity by 15% to its minimum value.