Homework #6

Reading

- Kasap 4.5-4.7

Problems

1. Work Functions

The work function of a metal Φ represents the energy needed to eject an electron from the Fermi level to the vacuum level. However, this energy depends on the crystal plane from which the electron is ejected. Additionally, surface conditions such as a thin oxide layer or contaminants on the surface would modify the observed Φ. Measurements of Φ are therefore done under high vacuum condition on clean crystal surfaces. Table 4.6 lists measured Φ for single crystal and polycrystalline samples of Al, Au and Ag that have a cubic crystal structure (FCC). Φ has been obtained for three different planes in the case of single crystals.

Table 4.6 The work function Φ (in eV) of aluminum, gold and silver for single crystal and polycrystalline samples.

	Φ (eV)			
Sample	(100)	(110)	(111)	Polycrystalline
Aluminum (Al)	4.41	4.06	4.24	4.28
Gold (Au)	5.47	5.37	5.31	5.40
Silver (Ag)	4.64	4.52	4.74	4.65

Data from H.B. Michaelson, IBM J. Res. Develop., *22*, 72, 1978 and M. Uda et al, J. Electron. Spectrosc. Relat. Phenom., **88**, 643, 1998

a. For each material, what is the average Φ_{av} and mean standard deviation of the work function for the three planes of the single crystal samples?
b. For each material, what is the percentage difference between the Φ for the polycrystalline sample and Φ_{av} for the single crystal samples? Comment on the significance of the results.

2. Fermi Energy

Eqn. 4.22 in Kasap gives the Fermi energy (at 0 K) as $E_{F0} = \left(\frac{h^2}{8m_e}\right)\left(\frac{3n}{\pi}\right)^{2/3}$, where n is the conduction electron concentration. This is equivalent to the equation we derived in class. Kasap Eqn. 4.23 gives the Fermi energy as a function of temperature: $E_F = E_{F0}\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{E_{F0}}\right)^2\right]$.

a. If each atom contributes one conduction electron, what is the Fermi energy (in eV) of copper at 293 K?
b. Since this Fermi energy was derived from the Sommerfeld model, the energy is entirely kinetic energy (the potential energy in the infinite 3D box was zero). Thus, what is the speed v_F for conduction electrons in copper with energies near the Fermi energy calculated in part (a)?
c. If the conduction electrons in copper were treated as an ideal gas following Maxwell-Boltzmann statistics with $1/2kT$ per degree of freedom and 3 degrees of freedom, what would be their thermal speed v_{th} at 293 K?
d. Comment on the difference (or ratio) of v_F to v_{th} by comparing your results from parts (b) and (c). What fundamental quantum mechanical phenomena contribute to this difference?

3. Sommerfeld to Drude

In terms of the density of states $g(E_F)$, the general expression for the conductivity of metals at E_F given by

$$\sigma = \frac{1}{3} e^2 v_F^2 \tau g(E_F)$$

Show that within the free electron theory (Sommerfeld model), this reduces to $\sigma = e^2 n\tau/m_e$, the Drude expression.