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Homework #5 - Solution

1. Free Electron

Consider the one-dimensional wavefunction for a free electron traveling in a region where $V = 0$.

$$\psi = Ae^{ikx}$$

a. Write down the one-dimensional Hamiltonian for the free electron.

b. Using the given wavefunction, solve the time-independent Schrödinger equation to determine the energy eigenvalues.

c. Using the same wavefunction given above, repeat parts (a) and (b) for a free electron moving in a region where the potential is constant given by V_0.

d. Comparing the two situations, if the total energy E is the same in each case, what does that imply about the wavenumbers for the two situations? Are they the same?

Solution

(a)

Since $V = 0$, the time-independent SE is $H\psi = E\psi$, where H is the Hamiltonian and is given by

$$\boxed{H = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}}$$

(b)

Inserting the given wavefunction ψ into the SE, we find the energy eigenvalue as

$$\boxed{E = \frac{\hbar^2 k^2}{2m}}$$

(c)

Since $V = V_0$, the Hamiltonian becomes

$$\boxed{H = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_0}$$

Using the given wavefunction, the energy eignevalue is now

$$\boxed{E = \frac{\hbar^2 k^2}{2m} + V_0}$$

(d)

Now we can compare the situations in (a) and (b). The total energy E is the same, but they will have different wavenumbers, so we'll write k_a and k_b. Solving the above energy eigenvalues for the wavenumbers, we have

$$\boxed{k_a = \frac{\sqrt{2mE}}{\hbar} \qquad \text{and} \qquad k_b = \frac{\sqrt{2m(E - V_0)}}{\hbar}}$$

We can see that the wavenumber in the region with $V = V_0$ is reduced (assuming $V_0 > 0$). This means that the free electron is slower in the region where $V = V_0$. To see this more clearly, let's rearrange the wavenumber equations:

$$\hbar k_a = \sqrt{2mE} \qquad \text{and} \qquad \hbar k_b = \sqrt{2m(E - V_0)}$$

Since $\hbar k$ is the electron momentum, and momumentum is classically given by $p = mv$, we can see that the electron in the region with $V = V_0$ will have a smaller momentum and smaller velocity.

2. Infinite Potential Well

Consider an electron bound within a one-dimensional infinite potential well with a width $a = 0.11$ nm.

a. What is the ground state energy ($n = 1$) for the electron?

b. What is the energy required to excite the electron to the second energy level?

c. What is the wavelength (in nm) of a photon that could transfer enough energy to excite the electron from the ground state to the second energy level?

Solution

(a)

The energy levels within a one-dimensional potential well are given by

$$E_n = \frac{n^2 h^2}{8ma^2}$$

where n is an integer that specifies the energy level, h is Planck's constant, m is the electron mass, and a is the width of the well. Thus, for $n = 1$ and $a = 0.11$ nm, we have

$$E_1 = \frac{1^2(6.626 \times 10^{-34}\text{J} \cdot \text{s})^2}{8(9.11 \times 10^{-31}\text{kg})(0.11 \times 10^{-9}\text{m})^2} \qquad \Rightarrow \qquad \boxed{E_1 = 4.98 \times 10^{-18} \text{ J} = 31.1 \text{ eV}}$$

(b)

For $n = 2$, we have

$$E_2 = \frac{2^2(6.626 \times 10^{-34}\text{J} \cdot \text{s})^2}{8(9.11 \times 10^{-31}\text{kg})(0.11 \times 10^{-9}\text{m})^2} \quad \Rightarrow \quad \boxed{E_2 = 19.91 \times 10^{-18} \text{ J} = 124.47 \text{ eV}}$$

Thus, the energy difference between level 2 and level 1 is

$$\Delta E_{1\to2} \quad = \quad E_2 - E_1 \quad \Rightarrow \quad \boxed{\Delta E = 14.9 \times 10^{-18} \text{ J} = 93.4 \text{ eV}}$$

(c)

The energy of a photon of wavelength λ is given by

$$E_{ph} = \frac{hc}{\lambda}$$

Rearranging to solve for wavelength, we have

$$\lambda = \frac{hc}{E_{ph}} \quad \Rightarrow \quad \lambda = \frac{1.986 \times 10^{-25} \text{ J} \cdot \text{m}}{E_{ph}} \quad \text{or} \quad \frac{1241.5 \text{ eV} \cdot \text{nm}}{E_{ph}}$$

With the difference in energy levels calculated above, using either Joules or electron volts, we find

$$\boxed{\lambda = 13.3 \times 10^{-9} \text{ m} = 13.3 \text{ nm}}$$

13.3 nm is in the "extreme ultraviolet" (EUV) portion of the electromagnetic spectrum (also called soft X-rays). EUV light at 13.5 nm generated by the relaxation of Sn ions is being used to create semiconductor devices through EUV photolithography.