Homework #2

Due Tuesday September 6 by noon

Reading

- Chapter 2.1-2.4

Videos

- Drude Model
- Resistivity and Mobility

Problems

1. (2.2) **Electrical Conduction**

The resistivity of aluminum at 25 °C has been measured to be 2.72×10^{-8} Ω m. The thermal coefficient of resistivity of aluminum at 0 °C is 4.29×10^{-3} K^{-1}. Aluminum has a valency of 3, a density of 2.70 g cm^{-3}, and an atomic mass of 27.

a. Calculate the resistivity of aluminum at -40 ºC.

b. What is the thermal coefficient of resistivity at -40 ºC?

c. Using the resistivity, estimate the mean free time between collisions for the conduction electrons in aluminum at 25 °C, and then estimate their drift mobility.

d. If the mean speed of the conduction electrons is about 2×10^6 m s^{-1}, calculate the mean free path and compare this with the size of a unit cell in Al (Al is FCC with a lattice parameter of 0.40478 nm). What should be the thickness of an Al film that is deposited on an IC chip such that its resistivity is the same as that of bulk Al?

e. What is the percentage change in the power loss due to Joule heating of the aluminum wire when the temperature drops from 25 °C to -40 ºC?

2. (2.5) **Effective number of conduction electrons per atom**

a. Electron drift mobility in tin (Sn) is 3.9 cm^2 V^{-1} s^{-1}. The room temperature (20 °C) resistivity of Sn is about 110 nΩ m. Atomic mass M_{at} and density of Sn are 118.69 g mol^{-1} and 7.30 g cm^{-3}, respectively. How many "free" electrons are donated by each Sn atom in the crystal? How does this compare with the position of Sn in Group IVB of the Periodic Table?

b. Consider the resistivity of few selected metals from Groups I to IV in the Periodic Table in Table 2.8 below. Calculate the number of conduction electrons contributed per atom and compare this with the location of the element in the Periodic Table. What is your conclusion?

Table 2.8 Selection of metals from Groups I to IV in the Periodic Table

Metal	Periodic Group	Valency	Density (g cm^{-3})	Resistivity (nΩ m)	Mobility (cm^2 V^{-1}s^{-1})
Na	IA	1	0.97	42.0	53
Mg	IIA	2	1.74	44.5	17
Ag	IB	1	10.5	15.9	56
Zn	IIB	2	7.14	59.2	8
Al	IIIB	3	2.7	26.5	12
Sn	IVB	4	7.30	110	3.9
Pb	IVB	4	11.4	206	2.3

NOTE: Mobility from Hall-effect measurements.

3. (2.11) **Resistivity of solid solution metal alloys: testing Nordheim's rule**

Nordheim's rule accounts for the increase in the resistivity from the scattering of electrons from the random distribution of impurity (solute) atoms in the host (solvent) crystal. Table 2.12 below lists some solid solution metal alloys and gives the resistivity ρ at one composition X and asks for a prediction ρ' based on Nordheim's rule at another composition X'. Fill in the table for predicted ρ' and compare (i.e., compute the percent difference) the predicted values with the experimental values, and comment.

Table 2.12 Resistivities of some solid solution metal alloys

	Alloy							
	Ag-Au	Au-Ag	Cu-Pd	Ag-Pd	Au-Pd	Pd-Pt	Pt-Pd	Cu-Ni
X (at.%)	8.8% Au	8.77% Ag	6.2% Pd	10.1% Pd	8.88% Pd	7.66% Pt	7.1% Pd	2.16% Ni
ρ_0 (nΩ m)	16.2	22.7	17	16.2	22.7	108	105.8	17
ρ at X (nΩ m)	44.2	54.1	70.8	59.8	54.1	188.2	146.8	50
C_{eff}								
X'	15.4% Au	24.4% Ag	13% Pd	15.2% Pd	17.1% Pd	15.5% Pt	13.8% Pd	23.4% Ni
ρ' at X'(nΩ m)								
ρ' at X'(nΩ m) Experimental	66.3	107.2	121.6	83.8	82.2	244	181	300
Percentage Difference								

Table 2.12 NOTE: First symbol (e.g., Ag in AgAu) is the matrix (solvent) and the second (Au) is the added solute. X is in at.%, converted from traditional weight percentages reported with alloys. C_{eff} is the effective Nordheim coefficient in $\rho = \rho_0 + C_{eff} X(1-X)$.

4. (2.20) **Ag–Ni alloys (contact materials) and the mixture rules**

   Silver alloys, particularly Ag alloys with the precious metals Pt, Pd, Ni, and Au, are used extensively as contact materials in various switches. Table 2.15 below shows the measured resistivities of four Ag–Ni alloys used in make-and-break and disconnect contacts with current ratings up to ~100 A.

   a. Ag–Ni is a two-phase alloy, a mixture of Ag-rich and Ni-rich phases. Using an appropriate mixture rule (i.e., Eqn. 2.34 in Kasap 4th Ed.), predict the resistivities of the alloys. Explain the differences between the predicted and experimental values.

   b. Compare the resistivity of Ag–10% Ni with that of Ag–10% Pd in Table 2.12 above. The resistivity of the Ag–Pd alloy is almost a factor of 3 greater. Ag–Pd is an isomorphous solid solution, whereas Ag–Ni is a two-phase mixture. Explain the difference in the resistivities of Ag–Ni and Ag–Pd.

Table 2.15 Resistivity of Ag-Ni contact alloys for switches

	Ni % in Ag-Ni alloy						
	0	10	15	20	30	40	100
d (g cm^{-3})	10.49	10.25	10.15	10.05	9.8	9.7	8.91
ρ (nΩ m)	16.9	18.7	19.0	20.0	24.4	27.0	71.0

NOTE: Compositions are in wt.%. Ag–10% Ni means 90% Ag–10% Ni. d = density and ρ = resistivity. Use volume fraction of Ni = $w_{Ni}(d_{alloy}/d_{Ni})$, where w_{Ni} is the Ni weight fraction, to convert wt.% to volume %. Data combined from various sources.